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ADDENDUM

Coupling coefficients of *SO*(*n*) and integrals involving Jacobi and Gegenbauer polynomials

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Abstract

A new and short proof of expression (3.2e) of Ališauskas (2002 *J. Phys. A: Math. Gen.* **35** 7323) with obvious triangle conditions for integrals of the Jacobi polynomials is proposed, without any allusion to the special isofactors of $Sp(4)$. Expressions (3.6c) and (3.12) are corrected.

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Recently in section 3 of [1], the integrals involving triplets of the Gegenbauer and the Jacobi polynomials and corresponding to special coupling coefficients of $SO(n)$ have been rearranged, using their relation with the semistretched isofactors of the second kind for the complementary chain $Sp(4) \supset SU(2) \times SU(2)$.

In contrast to the possible proof of identities (3.2c)–(3.2e) of [1] by a direct but not obvious transformation procedure discussed in the concluding remarks (section 7) of [1], expression (3.2d) of [1] for the integrals involving the product of three Jacobi polynomials may be rearranged straightforwardly, without any allusion to the special isofactors of $Sp(4)$. For this purpose we apply the symmetry relation (3.2a)–(3.2b) of [1] (i.e. interchange α_a and β_a, $a = 0, 1, 2, 3$) to (3.2d). When α_0 and β_0 are integers, the ${}_3F_2(1)$ type sums over z_i in the (modified) expressions (3.2d) and (3.2e) of [1] correspond to the Clebsch–Gordan coefficients of $SU(2)$ with the equivalent Regge 3×3 symbols

$$\left\| \begin{matrix} k_i & k_i + \alpha_i + \beta_i & p_i' - z_j - z_k \\ p_i' + \beta_i + k_j + k_k - z_j - z_k & p_i'' & k_i + \alpha_i \\ p_i'' + \alpha_i & p_i' + k_j + k_k - z_j - z_k & k_i + \beta_i \end{matrix} \right\| \tag{1}$$

and

$$\left\| \begin{matrix} p_i - z_j - z_k & k_i & k_i + \alpha_i + \beta_i \\ k_i + \beta_i & p_i'' + \alpha_i & p_i' + k_j + k_k - z_j - z_k \\ k_i + \alpha_i & p_i' + \beta_i + k_j + k_k - z_j - z_k & p_3'' \end{matrix} \right\| \tag{2}$$

expressed in both cases by means of (15.1c) of Jucys and Bandzaitis [2] (see also (7) of section 8.2 of [3]), but with hidden triangular conditions in the first case. For possible non-integer values of α_0 and/or β_0, the doubts as to the equivalence of these finite ${}_3F_2(1)$ series may be caused by the absence of mutually coinciding integer parameters (k_i in the modified expression (3.2d) and $\min(p_i - z_j - z_k, p_i'')$ as triangular conditions in (3.2e), respectively) restricting summation over z_i, unless equation (15.1d) of [2] (together with possible inversion of summation) is used for the CG coefficient of $SU(2)$ with Regge symbol (2). Note that the proof of relation between the corresponding finite ${}_3F_2(1)$ series in (3.2d) and (3.2e) based on the composition of Thomae's transformation formulae or their Whipple's specifications for single restricting parameter (see [4, 5]) is rather complicated.

The denominator factor $(l_i' + n/2)_{z_i}$ in (3.6c) of [1] should be corrected to $(l_i + n/2)_{z_i}$, and the denominator factor $(\alpha_i + 3/2)_{z_i}$ in (3.12) of [1] should be corrected to $(\alpha_i + k_i + 3/2)_{z_i}$. The factor $C_{l_i - l'}^{l'+n/2-1}(\cos\theta)$ in (3.8a) of [1] should be replaced (twice) by $C_{l_1 - l'}^{l'+n/2-1}(\cos\theta)$.

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